MATHEMATICS - I

(Calculus and Linear Algebra)

For Computer Science Engineering Branches

Reena Garg

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ISBN: 978-93-91505-27-1
Book Code: UG012EN

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by Reena Garg
[English Edition]

First Edition: 2021

Published by:
Khanna Book Publishing Co. (P) Ltd.
Visit us at: **www.khannabooks.com**
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CIN: U22110DL1998PTC095547

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Printed in India.

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FOREWORD

Engineering has played a very significant role in the progress and expansion of mankind and society for centuries. Engineering ideas that originated in the Indian subcontinent have had a thoughtful impact on the world.

All India Council for Technical Education (AICTE) had always been at the forefront of assisting Technical students in every possible manner since its inception in 1987. The goal of AICTE has been to promote quality Technical Education and thereby take the industry to a greater heights and ultimately turn our dear motherland India into a Modern Developed Nation. It will not be inept to mention here that Engineers are the backbone of the modern society - better the engineers, better the industry, and better the industry, better the country.

NEP 2020 envisages education in regional languages to all, thereby ensuring that each and every student becomes capable and competent enough and is in a position to contribute towards the national growth and development.

One of the spheres where AICTE had been relentlessly working from last few years was to provide high-quality moderately priced books of International standard prepared in various regional languages to all it's Engineering students. These books are not only prepared keeping in mind it's easy language, real life examples, rich contents and but also the industry needs in this everyday changing world. These books are as per AICTE Model Curriculum of Engineering & Technology – 2018.

Eminent Professors from all over India with great knowledge and experience have written these books for the benefit of academic fraternity. AICTE is confident that these books with their rich contents will help technical students master the subjects with greater ease and quality.

AICTE appreciates the hard work of the original authors, coordinators and the translators for their endeavour in making these Engineering subjects more lucid.

(Anil D. Sahasrabudhe)

Acknowledgement

The author grateful to AICTE for their meticulous planning and execution to publish the technical book for Engineering and Technology students.

I sincerely acknowledge the valuable contributions of the reviewer of the book Prof. Garima Singh, for making it students' friendly and giving a better shape in an artistic manner.

This book is an outcome of various suggestions of AICTE members, experts and authors who shared their opinion and thoughts to further develop the engineering education in our country.

It is also with great honour that I state that this book is aligned to the AICTE Model Curriculum and in line with the guidelines of National Education Policy (NEP) -2020. Towards promoting education in regional languages, this book is being translated in scheduled Indian regional languages.

Acknowledgements are due to the contributors and different workers in this field whose published books, review articles, papers, photographs, footnotes, references and other valuable information enriched us at the time of writing the book.

Finally, I like to express our sincere thanks to the publishing house, M/s. Khanna Book Publishing Company Private Limited, New Delhi, whose entire team was always ready to cooperate on all the aspects of publishing to make it a wonderful experience.

Reena Garg

Preface

Mathematics is a necessary avenue to scientific knowledge which opens new vistas of mental ability. Engineering mathematics offers a balance of theory and practice, which is intellectually stimulating. Learning the craft of applying mathematics to real world problems allow an Engineering student to find the solutions of the problem.

Calculus and Linear Algebra is intended mainly for undergraduate students of B.Tech **(CSE)** of 21st century with the aim to provide a sound understanding in the subject of mathematics.. This book is strictly aligned with AICTE model curriculum incorporating student centric and self-learning activities as per New National Education Policy based on **OBE** and **Bloom Taxonomy**. The topics are well organized to create interest among readers to study and apply the mathematical tools in engineering and science disciplines. The book mainly emphasizes on the practical applications of the concepts discussed in the units which will help the students to incorporate a deliberate focus on problem - solving skills.

The book consists of **5** units. For more understanding of the topic, a good number of relatively competitive problems are given at the end of each unit in the form of **short questions, HOTS, assignments, MCQs** and **know more**. **Practical/Projects/Activity** also given in each unit for enhancing the student's capability and to increase the feeling of team work. To clarify the subject, the text has been supplemented through **Notes, Observations** and **Remarks**. An attempt has been made to explain the topics through maximum use of geometries wherever possible.

Unit-1 deals with the application of derivatives, curvature, definite and improper integrals, Beta-Gamma functions with their properties,

Unit-2 is concerned to find the solution by using Rolle's theorem, Mean value theorem, Taylor's and Maclaurin's theorems, L'Hospital Rule and Maxima-minima for one variable.

Unit-3 deals with matrices, determinant, solution of linear system of equations with various methods, rank, Crammer's Rule, Gauss Elimination method and Gauss Jordan method with examples.

Unit-4 focuses on vector space, dependence, independence of vectors, basis, dimension, Inverse of a linear transformation, rank- nullity theorem, composition of linear maps with matrix associated with it.

Unit-5 discusses eigen values, eigenvectors, diagonalization, Inner product spaces, Gram-Schmidt orthogonalization and theorems based of symmetric and skew-symmetric matrices.

Mathematics is a subject that can be mastered only through hard work and practice. Practice is the only key word in the learning process of mathematics.

I hope this book will meet the requirements and expectations of all the engineering students. Although every care has been taken to avoid misprints and mistakes, yet it is difficult to claim perfection. I will gratefully receive and acknowledge every comment and suggestions from the teachers and the students leading to improvements in the text as well as in solved examples.

Reena Garg

Outcome Based Education

For the implementation of an outcome based education the first requirement is to develop an outcome based curriculum and incorporate an outcome based assessment in the education system. By going through outcome based assessments evaluators will be able to evaluate whether the students have achieved the outlined standard, specific and measurable outcomes. With the proper incorporation of outcome based education there will be a definite commitment to achieve a minimum standard for all learners without giving up at any level. At the end of the programme running with the aid of outcome based education, a student will be able to arrive at the following outcomes:

PO-1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

PO-2. **Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

PO-3. **Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

PO-4. **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

PO-5. **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

PO-6. **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

PO-7. **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

PO-8. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

PO-9. **Individual and team work:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

PO-10. Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

PO-11. Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

PO-12. Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Course Outcomes

After completion of the course the students will be able to:

CO-1: Apply Differential and Integral Calculus to notion of curvature, Centre of curvature and evaluate improper integrals using correct mathematical limit notation. Apart from these applications they will have a basic understanding of Beta and Gamma Functions

CO-2: Examine the behaviour of function for a given interval and expansion of trigonometric and transcendental functions

CO-3: Formulate, analyse, solve and apply the concept of matrices on the problems based on linear system of equations and relate them with linear transformations.

CO-4: Classify linear Independence and linear dependence of vectors and explain the concepts of rank, basis and dimension of vector Space, in addition of this, also learn to composition of linear maps and association with matrices.

CO-5: Apply essential tool to solve numerical problems based on Eigen values, Eigen vectors, Eigenbases, diagonalisation and orthogonalisation with the help of, linear algebra. Also deal with various properties of Eigen values which are used to solve many complex problems in various branches of engineering. In addition to that aware with the concept of norm of a vector , orthonormal and orthogonal vectors

Mapping of Course Outcomes with Programme Outcomes to be done according to the matrix given below:

Course Outcome	Expected Mapping with Programme Outcomes *(1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation)*											
	PO-1	**PO-2**	**PO-3**	**PO-4**	**PO-5**	**PO-6**	**PO-7**	**PO-8**	**PO-9**	**PO-10**	**PO-11**	**PO-12**
CO-1	3	2	2	1	1	-	2	-	-	-	-	-
CO-2	3	2	2	2	-	-	-	-	-	-	-	1
CO-3	3	3	3	1	2	2	-	-	1	1	-	1
CO-4	3	2	1	1	1	1	-	-	-	-	-	-
CO-5	3	2	2	2	2	1	-	-	-	-	1	-

Abbreviations and Symbols

SYMBOLS AND FORMULAE

1. Number System

N – set of natural numbers
$\mathbb{Z}$ – set of integers
Q – set of rational numbers
I – set of irrational numbers
$\mathbb{R}$ – set of real numbers
C – set of complex numbers
R^n – set of n-tuples

2. Greek Letters

α – alpha
β – beta
γ – gamma
Γ – capital gamma
δ – delta
Δ – capital delta
ε – epsilon
ι – iota
θ – theta
λ – lambda
μ – mu
ϕ – phi
ψ – psi
η – eta
π – pi
ρ – rho
κ – kappa

3. Notation in sets

$\in$ – belongs to
$\notin$ – not belongs to
$\cup$ – Union
$\cap$ – Intersection
( ) – open interval
[ ] – close interval
$\subseteq$ – subset
$\nsubseteq$ – not subset
$\subset$ – proper subset
$\not\subset$ – not a proper subset
$\supset$ – superset
{ } – set
ϕ – empty set
$<$ – strictly less than
$>$ – strictly greater than
$\leq$ – less than or equal to
$\geq$ – greater than or equal to

4. Some Other Useful Symbols

$\sim$ – equivalent to
$\leftrightarrow$ – interchange
∞ – infinity
$\int$ – integral
! – factorial
$\Rightarrow$ – implies
$\forall$ – for all
$\Leftrightarrow$ – implies and implied by
$\| \ \|$ – norm
$| \ |$ – modulus
: – colon
; – semicolon
$[A : B]$ or $[A/B]$ – Augmented Matrix

5. Nature of Roots of an Quadratic equations

If $ax^2 + bx + c = 0$ is quadratic, then

(*a*) its roots are given by $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(*b*) the sum of the roots is equal to $-b/a$

(*c*) product of the roots is equal to c/a

(*d*) $b^2 - 4ac = 0 \Rightarrow$ the roots are equal

(*e*) $b^2 - 4ac > 0 \Rightarrow$ the roots are real and distinct

(*f*) $b^2 - 4ac < 0 \Rightarrow$ the roots are complex

(*g*) If $b^2 - 4ac$ is a perfect square, then the roots are rational.

6. Properties of Logarithm

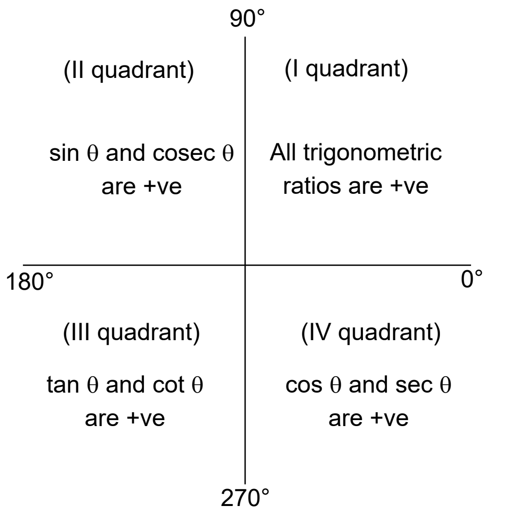
(*a*) $\log_a 1 = 0, \log_a 0 = -\infty$ for $a > 1$,
$\log_a a = 1$
$\log_e 2 = 0.6931$
$\log_e 10 = 2.3026, \log_{10} e = 0.4343$

(*b*) $\log_a p + \log_a q = \log_a pq$

(*c*) $\log_a p - \log_a q = \log_a \dfrac{p}{q}$

(*d*) $\log_a p^q = q \log_a p$

7. Nature of Trigonometric Ratios in Quadrant



8. Product and Sum Formulae for trigonometric functions

(*a*) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

(*b*) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

(*c*) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

(*d*) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

(*e*) $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(*f*) $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

(*g*) $\sin 2A = 2 \sin A \cos A = \dfrac{2 \tan A}{1 + \tan^2 A}$

(*h*) $\cos 2A = \cos^2 A - \sin^2 A$
$= 1 - 2 \sin^2 A$
$= 2 \cos^2 A - 1 = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

(*i*) $\tan 2A = \dfrac{\sin 2A}{\cos 2A} = \dfrac{2 \tan A}{1 - \tan^2 A}$

(*j*) $\sin 3A = 3 \sin A - 4 \sin^3 A$

(*k*) $\cos 3A = 4 \cos^3 A - 3 \cos A$

(*l*) $\tan 3A = \dfrac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

(*m*) $\sin A + \sin B = 2 \sin \dfrac{A+B}{2} \cos \dfrac{A-B}{2}$

(*n*) $\sin A - \sin B = 2 \cos \dfrac{A+B}{2} \sin \dfrac{A-B}{2}$

(*o*) $\cos A + \cos B = 2 \cos \dfrac{A+B}{2} \cos \dfrac{A-B}{2}$

(*p*) $\cos A - \cos B = 2 \sin \dfrac{A+B}{2} \sin \dfrac{B-A}{2}$

(*q*) $\sin A \cos B = \dfrac{1}{2} [\sin(A+B) + \sin(A-B)]$

(*r*) $\cos A \sin B = \dfrac{1}{2} [\sin(A+B) - \sin(A-B)]$

(*s*) $\cos A \cos B = \dfrac{1}{2} [\cos(A+B) + \cos(A-B)]$

(*t*) $\sin A \sin B = \dfrac{1}{2} [\cos(A-B) - \cos(A+B)]$

(*u*) $\sin x = 0 \Leftrightarrow x = n\pi, n \in Z$

(*v*) $\sin x = \pm 1 \Leftrightarrow x = (4n \pm 1) \dfrac{\pi}{2}, n \in Z$

(*w*) $\cos x = 0 \Leftrightarrow x = (2n + 1) \dfrac{\pi}{2}, n \in Z$

(*x*) $\cos x = \pm 1 \Leftrightarrow x = 2n\pi$ and $x = (2n + 1)\pi$, $n \in Z$

(*y*) $e^{ax} \neq 0, \forall x \in R; a \in R$

9. Basic differentiation formulae

(*a*) $\dfrac{d}{dx} (\sin x) = \cos x$

(*b*) $\dfrac{d}{dx} (\cos x) = -\sin x$

(*c*) $\dfrac{d}{dx} (\tan x) = \sec^2 x$

(d) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

(e) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(f) $\frac{d}{dx}(\text{cosec}\, x) = -\text{cosec}\, x \cot x$

(g) $\frac{d}{dx}(e^x) = e^x$

(h) $\frac{d}{dx}(a^x) = a^x \log_e a$

(i) $\frac{d}{dx}(\log_a x) = \frac{1}{x \log a}$

(j) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

(k) $\frac{d}{dx}(ax+b)^n = na(ax+b)^{n-1}$

(l) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, x \neq \pm 1$

(m) $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}, x \neq \pm 1$

(n) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

(o) $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

(p) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}, x \neq 0, \pm 1$

(q) $\frac{d}{dx}(\text{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}, x \neq 0, \pm 1$

(r) $\frac{d}{dx}(\sin hx) = \cos hx$

(s) $\frac{d}{dx}(\cos hx) = -\sin hx$

10. Basic Integration Formulae

(a) $\int \sin x\, dx = -\cos x + c$

(b) $\int \cos x\, dx = \sin x + c$

(c) $\int \tan x\, dx = -\log \cos x + c = \log \sec x + c$

(d) $\int \cot x\, dx = \log \sin x + c$

(e) $\int \sec x\, dx = \log(\sec x + \tan x) + c$

(f) $\int \text{cosec}\, x\, dx = \log(\text{cosec}\, x - \cot x) + c$

(g) $\int \sec^2 x\, dx = \tan x + c$

(h) $\int \text{cosec}^2 x\, dx = -\cot x + c$

(i) $\int e^x dx = e^x$

(j) $\int a^x dx = \frac{a^x}{\log_e a} + c; a > 0, a \neq 1$

(k) $\int \frac{1}{x} dx = \log_e x + c$

(l) $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$

(m) $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

(n) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right) + c$

(o) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right) + c$

(p) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$

(q) $\int \frac{dx}{\sqrt{a^2+x^2}} = \sin h^{-1}\frac{x}{a} + c$

(r) $\int \frac{dx}{\sqrt{x^2-a^2}} = \cos h^{-1}\frac{x}{a} + c$

(s) $\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx)$

(t) $\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2+b^2}(a \cos bx + b \sin bx)$

ABBREVIATIONS

lim	–	limit
$\therefore$	–	therefore
$\because$	–	because of
i.e.,	–	that is
$f^n(a)$	–	*n*th derivative of (f) at 'a'
sup.	–	supremum
Inf.	–	infimum
$Lf'(a)$	–	left hand derivative of 'f' at 'a'
$Rf'(a)$	–	right hand derivative of 'f' at 'a'
$Lf(a)$	–	left hand limit of 'f' at 'a'
$Rf(a)$	–	right hand limit of 'f' at 'a'
diag.	–	diagonal
L.H.S.	–	left hand side
R.H.S.	–	right hand side
dim	–	dimension
adj (A)	–	adjoint of matrix A
min.	–	minimum
max.	–	maximum
L.C.	–	linear combination
L.D.	–	linear dependence
L.I.	–	linear independence

List of Figures

Unit 1: Calculus I

Unit 2: Calculus II

Unit 3: Matrices

Guidelines for Teachers

To implement Outcome Based Education (OBE) knowledge level and skill set of the students should be enhanced. Teachers should take a major responsibility for the proper implementation of OBE. Some of the responsibilities (not limited to) for the teachers in OBE system may be as follows:

- Within reasonable constraint, they should manipulate time to the best advantage of all students.
- They should assess the students only upon certain defined criterion without considering any other potential ineligibility to discriminate them.
- They should try to grow the learning abilities of the students to a certain level before they leave the institute.
- They should try to ensure that all the students are equipped with the quality knowledge as well as competence after they finish their education.
- They should always encourage the students to develop their ultimate performance capabilities.
- They should facilitate and encourage group work and team work to consolidate newer approach.
- They should follow Blooms taxonomy in every part of the assessment.

Bloom's Taxonomy

Level	Teacher should Check	Student should be able to	Possible Mode of Assessment
Creating	Students ability to create	Design or Create	Mini project
Evaluating	Students ability to Justify	Argue or Defend	Assignment
Analysing	Students ability to distinguish	Differentiate or Distinguish	Project/Lab Methodology
Applying	Students ability to use information	Operate or Demonstrate	Technical Presentation/ Demonstration
Understanding	Students ability to explain the ideas	Explain or Classify	Presentation/Seminar
Remembering	Students ability to recall (or remember)	Define or Recall	Quiz

Guidelines for Students

Students should take equal responsibility for implementing the OBE. Some of the responsibilities (not limited to) for the students in OBE system are as follows:

- Students should be well aware of each UO before the start of a unit in each and every course.
- Students should be well aware of each CO before the start of the course.
- Students should be well aware of each PO before the start of the programme.
- Students should think critically and reasonably with proper reflection and action.
- Learning of the students should be connected and integrated with practical and real life consequences.
- Students should be well aware of their competency at every level of OBE.

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